Identification of Near-Surface Karst Cavities Using the Posterior Population Expansion Inverse Method Applied to Electrical Resistivity Data

Manon Trottet, Przemyslaw Juda, Arnulf Schiller, and Philippe Renard

Abstract

Traditional inverse methods used to interpret electrical resistivity tomography (ERT) measurements are not able to properly characterize karst environments due to the strong contrast between air or water filled conduits and the surrounding matrix. In fact, these inversion methods were originally conceived to interpret rather uniform lithologies. The posterior population expansion (PoPEx) method is an inverse method designed to deal with data presenting abrupt variations such as those encountered in a karst system. The advantage of this method is that it directly returns maps indicating the probabilities of encountering conduits instead of the usual deterministic set of resistivity values that one has to interpret manually. This method is tested here to invert synthetic 2D ERT data of karst systems. At this stage, PoPEx is computationally demanding and needs to be standardized. However, the results presented in this paper show that it is capable of properly identifying simple and synthetic caves. Further, research is needed to test its applicability in the field.

Keywords

Electrical resistivity tomography • Probabilistic inversion • Multiple-point statistics

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1 Introduction

Limestone environments are subject to significant karstification leading to the formation of large cavities. In many situations, it is necessary to be able to detect them as accurately as possible. For example, near-surface cavities can seriously impact any kind of construction through subsidence or collapse. Therefore, identifying them at an early stage may avoid important material and economical losses (Parise et al 2015). It may also be important to characterize the karst subsurface to locate, for example, where to drill new boreholes for the drinking water supply.

Geophysical techniques, and among them geoelectrical techniques, are broadly used methods to characterize efficiently the subsurface (Bechtel et al 2007; Chalikakis et al 2011). Although widely used and very effective in unconsolidated media, geoelectric methods are not fully effective in karst environments. In fact, locating the exact location of a conduit remains one of the most challenging tasks in karst research (Zhu et al. 2011).

Most traditional geophysical inversion methods are based on a deterministic approach and do not consider prior geological knowledge. Moreover, their most frequent numerical schemes assume that the spatial distribution of the petrophysical parameters (e.g., the electrical resistivity) is as smooth as possible. In the case of karstic systems, this can be an issue as conduits are discrete objects displaying strong contrasts with the underlying limestone.

Among the inverse methods that can deal with discrete problems is the posterior population expansion method (PoPEx) (Jäggli et al. 2017, 2018). This method was specifically developed to account for categorical distributions of the unknown parameters, i.e., the electrical resistivity when applied to ERT data. Moreover, it allows the integration in the inversion scheme of prior geological knowledge. Models of the possible distribution of the electrical resistivity values are defined before running the inversion.

B. Andreo et al. (eds.), *EuroKarst 2022, Málaga*, Advances in Karst Science,
https://doi.org/10.1007/978-3-031-16879-6_26

The method can be used for a broad range of applications in complex, geologically realistic, and discrete model space settings. Until now it has been tested on hydrogeological problems. This paper presents its first application on a geophysical data set. The study aims to investigate if the method can provide reasonable results when it is applied to interpret electrical resistivity tomography (ERT) data. The study is based on synthetic data sets for which the exact locations of the conduits are known and can be used to verify the quality of the results. Application to real field data is out of the scope of this paper and will be studied in future works.

2 Methods

2.1 The Posterior Population Expansion Method

All the mathematical theory underlying the PoPEx method is described in detail in Jäggli et al (2017) and Jäggli et al (2018). Here, for sake of brevity, only the main ideas are outlined. PoPEx is a generic probabilistic Bayesian inversion method. Its inversion scheme is based on a set of geological models that are generated using a geostatistical method and extended iteratively. At each iteration, the existing set of samples is used to learn, in a statistical sense, the relationship between the model parameters and the variables.

As for any Bayesian inversion technique, the first step is to define the statistical model of the prior distribution of the unknown parameter values (i.e., the electrical resistivity). This prior distribution expresses what we know about the underground before collecting the geophysical data. In karst systems, we will express the fact that we know that we expect to find cavities in limestone formations with a certain spatial geometry derived a priori from analogous sites and geological studies.

To express this knowledge in statistical terms, a multiple-point statistics (MPS) approach is used. Then, each geological model that is generated is transformed into petrophysical parameters. In this paper, to keep the problem very simple, we assumed that this relation is deterministic, but this is not an obligation.

The models are then used to compute the forward geophysical response which is compared to the observation data. This allows to define a misfit function and evaluate the likelihood by assuming a certain statistical distribution of the errors. Results are stored in an ensemble, and statistics from the ensemble are computed to check which features in the model space are likely to produce a good fit between the observations and the calculated data. This knowledge is then used to preferentially sample certain locations and parameter values and generate new models by conditioning the geostatistical simulations with this information. In this process, the algorithm progressively learns how to generate new models that are more likely to fit the data.

The PoPEx method generates a large ensemble of stochastic simulations and returns either the simulations having the highest probabilities of fitting the data or the probability maps of finding a conduit at a certain location.

2.2 Prior Parameter Distribution

To apply the PoPEX inversion method to ERT data, a statistical model of all the possible geometries of the encountered cavities is needed in order to define a prior parameters distribution. To this aim, the direct sampling (DS) MPS method (Mariethoz et al. 2010) is used.

This method is based on a training image (TI). The training image is a conceptual model that represents the general structures and the variability of the geometries that are expected to be found in the studied area. From the training image, the DS method learns the statistics of the pattern and produces stochastic simulations (realizations) that resemble the training image. The TI must be larger than the realizations so that the algorithm can deduce statistics. Figure 1a shows the training image used in this study. Size of the TI is 80 by 1000 m. Realizations of 15 m depth by 100 m long are produced from it. This TI is a concept derived from field observations and adapted to represent the expected variability of the geometry of karst conduits.

Figure 1b, c shows few examples of the many possible realizations obtained from the training image 1a with the DS algorithm. When no conditioning data is given, realizations can be very different and allow exploring a wide uncertainty space. If conditioning data, such as observations of the rock type at the surface or along a borehole are available, the simulations can account for that information (compare Fig. 1a, b).

2.3 Forward Model and Traditional Inversion

There are many codes and software available for inverting geophysical ERT data. Here, the pyGIMLi open-source python package (Rücker et al. 2017) is used for two different purposes.

Firstly, data acquisition is simulated through the forward ERT responses of the reference models to obtain sets of apparent resistivity values used as input for the inversion schemes. Responses are calculated using 72 electrodes

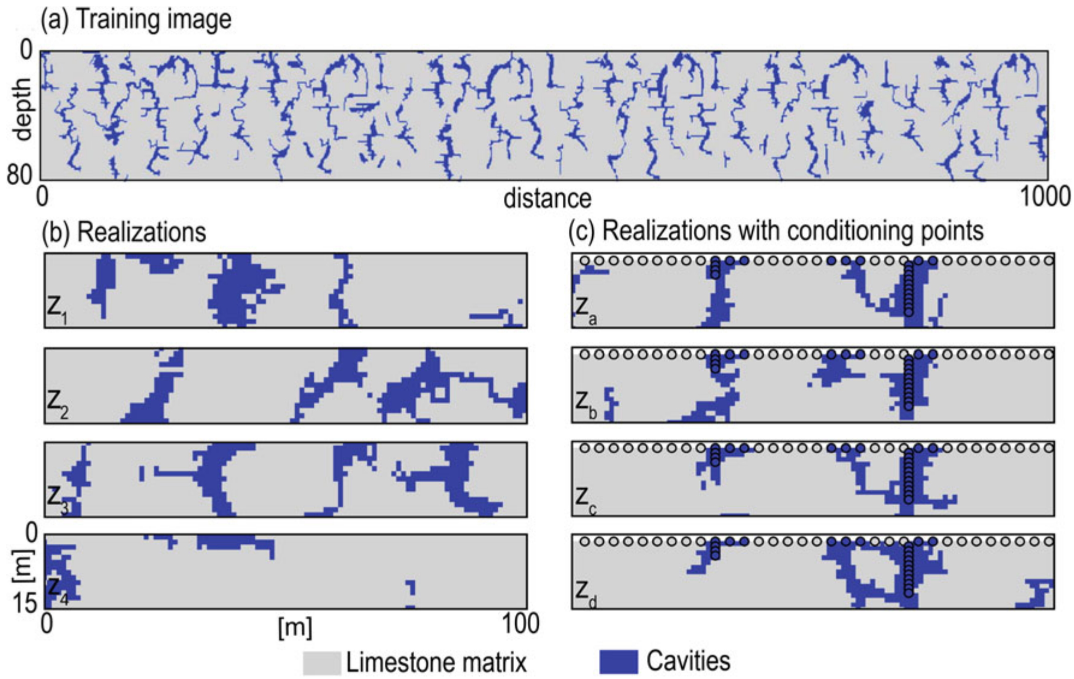

Fig. 1 Prior synthetic cross-sections that represent karst cavities. **a** is the training image. **b** is a set of four simulated realizations showing the variability that can be expected at the scale of this cross-section. **c** are a set of four realizations conditioned with observation points (the circles) located at the surface and along a borehole. In cases **c** the variability of the simulations is reduced as simulations are constrained by more data

regularly placed at the surface with a dipole–dipole setting. One example of the results of this forward computation is shown in Fig. 2. Secondly, a forward simulator is needed to run the PoPEx inversion. Finally, the standard inversion is also performed with pyGIMLi in order to compare its results with the results obtained with PoPEx.

3 Synthetic Case Study

In order to assess the PoPEx method, synthetic reference models are defined as the unknown truth of the numerical experiment. To this aim, three DS realizations (references 1, 2, and 3) were chosen (Fig. 3a). These references were selected because they show situations with different densities and complexities of conduits allowing to assess the performance of the methods in different situations.

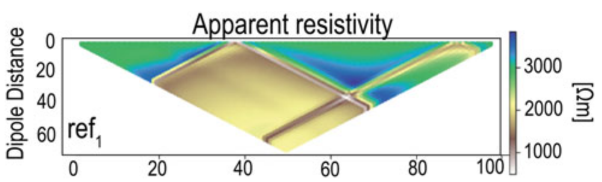

Fig. 2 Result of the forward ERT computation of Reference 1 using pyGIMLI

4 Results

4.1 Results Obtained with Standard Inversion Methods

Figure 3b shows the spatial distribution of the resistivity values obtained when a standard inversion is applied to the three reference models. For *reference 1*, we see a strong resistivity anomaly centered around the location of the main cavity and a smaller anomaly corresponding to the smaller cave close to the surface on the right side. In the second case, the anomalies are also located properly, but some artefacts are visible. Finally, for the denser and more complex geometry, it becomes much more difficult to relate the inverted resistivity fields to the exact geometry of the caves. In all cases, the exact geometry of the boundary of the caves is difficult to infer only based on the resistivity fields.

4.2 Results Obtained with PoPEx

Results of the PoPEx inversion are illustrated in Figs. 4 and 5. They are expressed either in terms of cross-sections showing the probability (between 0 and 1) of finding a cave at each location (Fig. 4) or as individual simulations

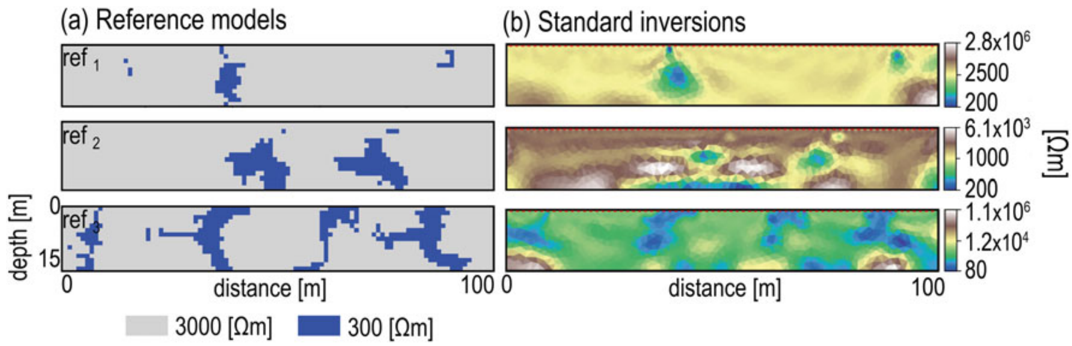

Fig. 3 Reference models and results of the standard inversion. **a** Three reference cross-sections, the cavities are represented in blue and the limestone matrix in grey. **b** Results of the standard inversion for these three reference cases

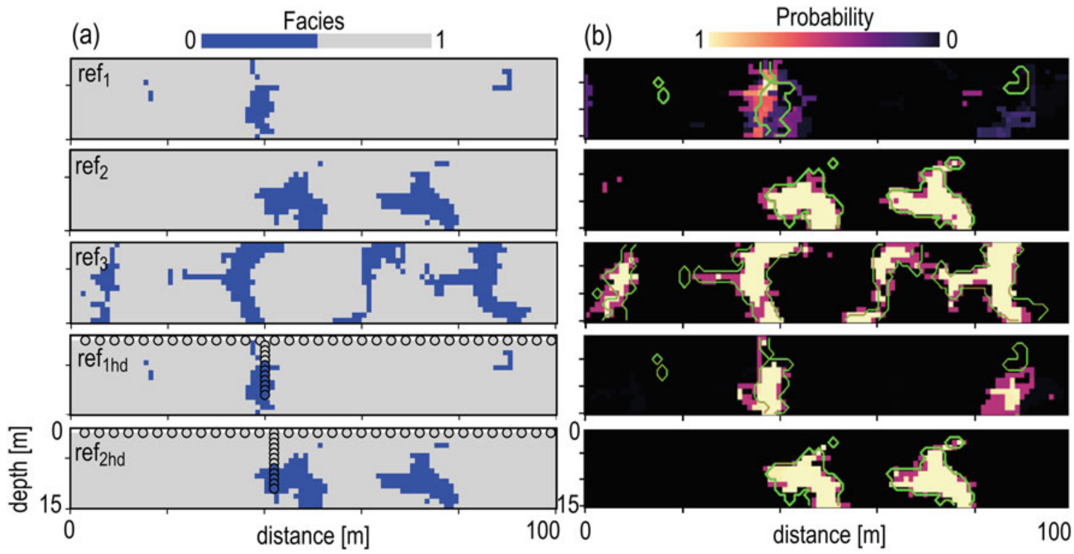

Fig. 4 Results of the PoPEx inversion. **a** Unknown references and **b** The resulting probabilities of the occurrence of a cave. The green lines represent the position of the true caves. The first three examples represent the same references as in Fig. 3. In addition, the last two examples illustrate how geological observations can be added to constrain the inversion and improve the results

(Fig. 5.). A probability of 1 means that there is 100% of chance to find limestone at this location.

Figure 4a shows the reference models. For references 1, 2, and 3, the inversion is performed without conditioning data, whereas references 1hd and 2hd include conditioning data at the surface and along a vertical line mimicking a possible borehole log. These conditioning points are shown with grey and blue circles in the reference models.

Figure 4b shows the probability maps obtained after the inversion for these models. A probability of 1 indicates that there is a 100% probability of finding a conduit at this location. The shapes of the true cavities are represented with a green line in the probability maps to facilitate the analysis of the results. Figure 4b shows that all the large and main cavities are detected and rather well represented in the probability maps. However, small cavities such as the one located in the upper right corner in *Reference 1* are not always detected properly.

In all the references, the uncertainty zones (zones with intermediate probabilities) are essentially occurring at the edges of the cavities. In most cases, the central part of the cavity is detected and delineated with a probability of 1, whereas outside of the cavities, the probability is correctly identified as 0. In between, intermediate probabilities are indicating some uncertainty on the exact position of the limit of the cave.

The only case in which the central part of the cave is not identified with a very high probability is the first example (*Reference 1*). In that case, the shape of the cavity is not well determined and seems to be slightly shifted from its actual position. It can be observed that adding conditioning data (*Reference 1hd*) improves significantly the quality of the predictions for the largest cave. The shape of the detected cavity is more accurate, and there are much more locations where the probability of finding a conduit is 1. However, in that case, artefacts appear, PoPEx identifies wrongly the position and the size of the small cavity on the right side. It finds that the cavity should be deeper than it is, and it is overconfident about the results giving a wrong probability of 1 instead of 0. For *Reference 2hd*, adding the conditioning data improves the results (without creating artefacts) only slightly because they were already good without conditioning data.

For *Reference 3,* results obtained with PoPEx (Fig. 4b) are much better than results obtained with the traditional inversion (Fig. 2b), even if the number of cavities is rather large.

Finally, Fig. 5b shows five of the individual simulations obtained for the five test cases. The PoPEx method generates a large ensemble of stochastic simulations, from which it produces the probability maps discussed above. Each simulation is associated with a posterior probability value. In Fig. 5b, the simulations having the highest probabilities are displayed. This figure shows that the geometries are close but slightly different from the true unknown reality (Fig. 5a).

An important point to notice with these individual realizations is that they can be used in a Monte Carlo procedure for probabilistic risk assessment studies. For example, it would be straightforward to use them as input in a geo-mechanical model to study how a construction project could be damaged if built on such type of ground. By repeating the mechanical computation with the different karst simulations, one could estimate for example the probability of collapse of the building from these results.

5 Discussion and Conclusion

The aim of the study was to assess if the PoPEx method, coupled with the direct sampling MPS algorithm, could be used to invert synthetic ERT geophysical data to detect near-surface karst cavities. Results showed that applied to simple and synthetic models, the method is able to detect the cavities, which shows the benefits of pursuing this strategy. Shape of the conduits is determined with a higher accuracy than with a classical inversion method.

An interesting feature of the approach is the possibility to integrate surface or borehole observations to condition the prior distribution of the electrical resistivity values and thus combine geological observations, geological concept and the geophysical data. The prior distribution of the electrical resistivity is performed using the MPS approach. One advantage of the MPS approach is its flexibility allowing to include a conceptual geological knowledge in the inversion process and generate rapidly many very different models. Here, only simple models where tested, but different levels of complexity could be easily studied. Indeed, it is rather easy to modify and increase step by step the complexity of the geological models: starting with very simple ones, as the examples presented above, and adding complexity by simply changing the training image.

Although results are very encouraging and PoPEx provides many advantages, results presented here must be taken

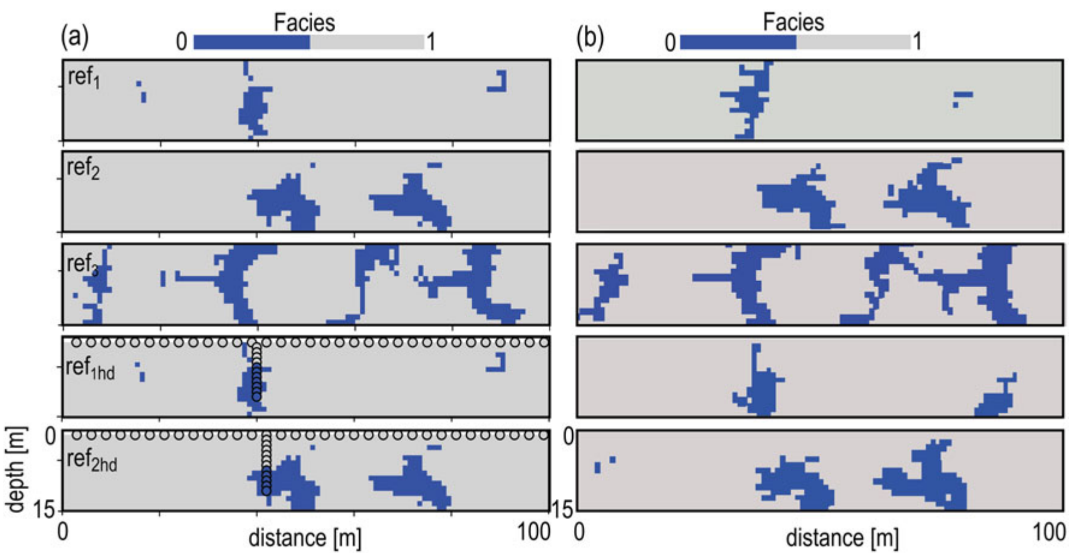

Fig. 5 Results of the PoPEx inversion. **a** The five unknown references and **b** the best individual simulations belonging to the posterior ensemble for the five different cases

with care. Two important points must be highlighted. First, results show low uncertainty in the probability maps. This behaviour could be due to the way the likelihood is computed. Different ways to compute it were tested and, a tempered likelihood was selected, but that aspect requires further sensitivity analysis. The second point is that the synthetic models used as references models were generated using the same conceptual geological model (the same training image) as the one used to define the prior distribution for the inversion. Reference models and the one used to define the prior distribution of the parameters are hence rather similar. Quality and robustness of the results still need to be evaluated if these images are different and if the training image does not contain enough variability to allow the MPS simulation algorithm to model the actual cave geometry. These cases must be further studied, especially when considering real field data. Another possible impact on the quality of the results is the variability of petrophysical parameters and the number of cavities in the rock matrix. This variability can be incorporated into the simulations, but its impact on the accuracy of the results must be tested.

To conclude, results showed that the PoPEx approach is able to improve the detection of cavities in simple and synthetic cases. The major drawbacks of the method are that it requires longer computing times than a standard inversion, and a conceptual model of the cave geometries needs to be defined. However, the results obtained with this method are very encouraging.

Acknowledgements The authors would like to thank A. Neven for his help in setting up the forward problem with pyGIMLi, J. Straubhaar for his support with the MPS model, as well as two anonymous reviewers who helped improve the paper. The project was funded by the Austrian Academy of Science via the FlowCast project.

References

Bechtel TD, Bosch FP, Gurk M (2007) Methods in karst hydrogeology, vol 26. CRC Press, Taylor & Francis Group; chap 9. Geophysical methods, pp 171–199

Chalikakis K, Plagnes V, Guerin R, Valois R, Bosch FP (2011) Contribution of geophysical methods to karst-system exploration: an overview. Hydrogeol J 19(6):1169–1180

Jäggli C, Straubhaar J, Renard P (2017) Posterior population expansion for solving inverse problems. Water Resour Res 53(4):2902–2916

Jäggli C, Straubhaar J, Renard P (2018) Parallelized adaptive importance sampling for solving inverse problems. Front Earth Sci 2018:203

Mariethoz G, Renard P, Straubhaar J (2010) The direct sampling method to perform multiple-point geostatistical simulations. Water Resour Res 46(11)

Parise M, Closson D, Gutierrez F, Stevanovic Z (2015) Anticipating and managing engineering problems in the complex karst environment. Environ Earth Sci 74(12):7823–7835

Rücker C, Günther T, Wagner FM (2017) pygimli: an open-source library for modelling and inversion in geophysics. Comput Geosci 109:106–123

Zhu J, Currens J, Dinger J (2011) Challenges of using electrical resistivity method to locate karst conduits a field case in the Inner Bluegrass Region, Kentucky. J Appl Geophys 75(3):523–530