Chi-Square Test

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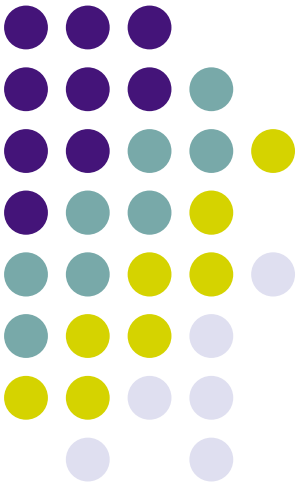
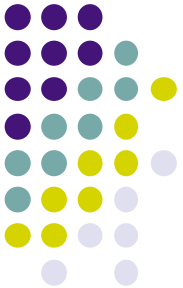


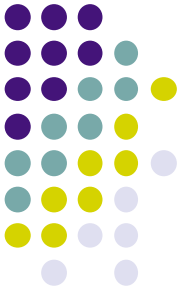
C. D. Manning & H. Schütze : *Foundations of statistical natural language processing*. The MIT Press. Cambridge (MA)

Discriminating Features



- How can we characterize / discriminate the distribution of a set of given word types (or other linguistic features) for corpus (or a document or a set of documents) in comparison with another?
  Compare two works of two different authors
- We can used word tokens, word types, bigrams, trigrams, phrases, POS, or even punctuations
- Used in various context
  - Parallel word-by-word translation
  - Pertinent collocations

Our US Corpus



- Speeches given by Senator Barack Obama

  150 speeches from Feb., 10th 2007
  420,410 tokens, 9,014 types

  For 2008 only: 113 speeches
  294,553 tokens, 7,663 types

  http://www.barackobama.com/

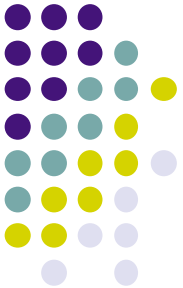
- Speeches given by Senator John McCain

  94 speeches. from Apr., 25th 2007
  206,899 tokens, 9,401 types

  For 2008 only: 71 speeches
  154,365 tokens, 7,792 types

  http://www.johnmccain.com/

Discriminating Features

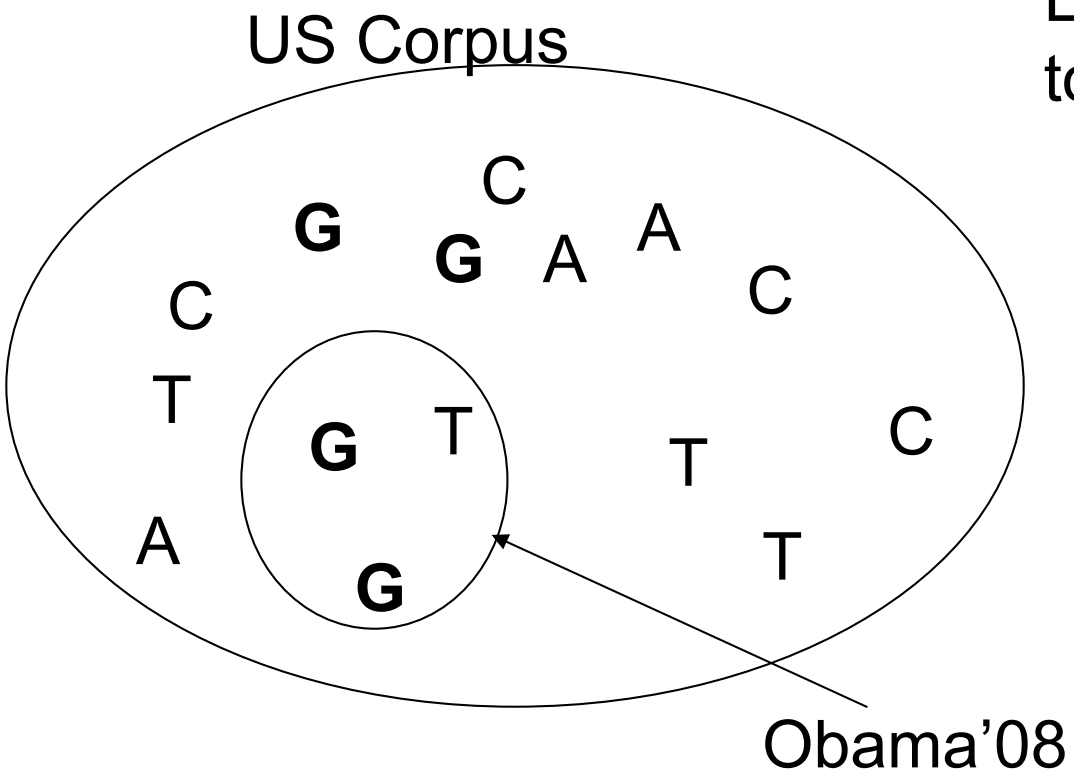


- To define whether a given feature (e.g., word, bigram, POS, etc.) is used significantly more often in a given corpus, we may subdivide the whole corpus (C) into two (or more) disjoint parts
- Example: US electoral speeches

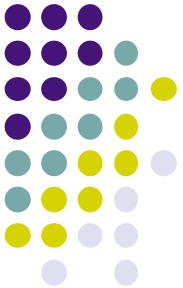
Our US Corpus

US: all speeches given by B. Obama & J. McCain during the years 2007 & 2008

Example with 15 tokens and 4 types



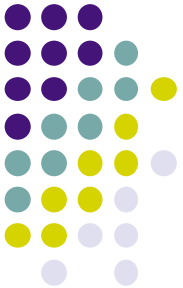
Contingency Table



- We can resume all needed information into a contingency table (one per word / feature)
- A large corpus **C** is subdivided into two (disjoint) parts **S** and **C-** (with **C** = **S** U **C-**)

	S	**C-**	
ω	*a*	*b*	*a* + *b*
not ω	*c*	*d*	*c* + *d*
	a + *c*	*b* + *d*	*n* = *a* + *b* + *c* + *d*

Contingency Table



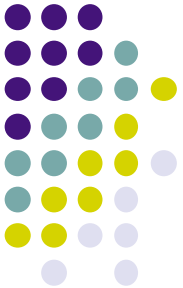
- Example

  The word “Bush” in McCain’s speeches in 2008 (**S**) vs. all US electoral Speeches (**C-**) (without **S**)

	S	**C-**	
“Bush”	26	398	424
not “Bush”	154,339	474,331	628,670
	154,365	474,729	629,094

- In the last column we have the value for the whole corpus **C**. E.g., the number of token “Bush” = 424.

Test Chi-Square

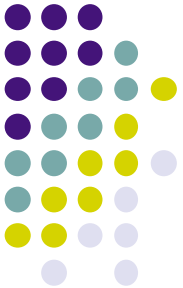


- Probability estimate
  Prob[“Bush” in **C**] = $(a+b)/n$ = 424/629,094 = 0.00067
  Prob[one word in **S**] = $(a+c)/n$ = 154,365/629,094 = 0.245

	S	**C-**	
“Bush”	26	398	424
not “Bush”	154,339	474,331	628,670
	154,365	474,729	629,094

- Does a significant relationship exist between the word type “Bush” and McCain’s speeches?
  (Does this distribution significantly differ between **S** and $\mathbf{C}_{8}$**-**?)

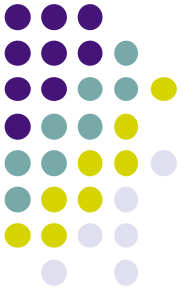
Test Chi-Square



- Distribution of four POS tags according to two authors
- Does this distribution differ significantly?
  Of course, we do not expect having the same values in both columns, but are the differences significant?

Observed	McCain'08	Obama'08	Total	Percentage
NN	33,876	58,550	92,426	41.6%
JJ	10,677	18,517	29,194	13.2%
VB	21,927	54,268	76,195	34.3%
RB	7,117	17,064	24,181	10.9%
Total	73,597	148,399	221,996	100%
Percentage	33.2%	66.8%		

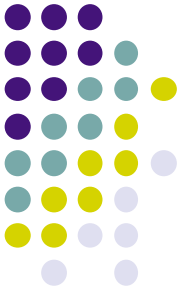
Test Chi-Square



- Each statistical test is based on a set of assumptions. For the chi-square test (or χ^2), we assume (we admit as truth that):

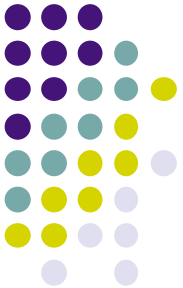
1. Each sample is a random sample
2. The samples are mutually independent
3. Each observation may be categorized into one of the *r* categories.

Test Chi-Square



- First we specify our null hypothesis (H_0):
  In our example, we assume that the use of one particular POS (for one word) by one author does not imply the use of a given POS (the same or another) by the other author.
  Under H_0, each author will use a similar number of each POS in his speeches (we admit random variations and thus we do not expect exactly the same values).
  If an author gives more speeches (or longer speeches), of course the number of each POS will increase but proportionally.

Test Chi-Square



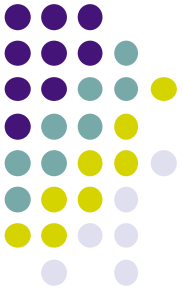
- Second, if the null hypothesis is not true, we must admit the (unique) alternate hypothesis (H_1).

  In our case, H_1 assume that there is a systematic difference in the POS distribution between the two authors.

  These two hypothesis cannot be true at the same time.
  Only one of them is true.
  Which one (according to the available data)?

- Third we compute the *expected* number of each POS according to each author under this null hypothesis (we do as if the null hypothesis H_0 is true)

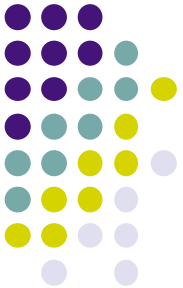
Test Chi-Square



For example, McCain produces 73,597 tokens and 41.6% must be nouns. Thus we expect 73,597 x 0.416 = 30,616.4 nouns. This value will be denoted E_i (and the observed value as O_i).

Expected	McCain'08	Obama'08	Percentage
NN			41.6%
JJ			13.2%
VB		50901	34.3%
RB			10.9%
Total	73,597	148,399	100%
Percentage	33.2%	66.8%	

Test Chi-Square

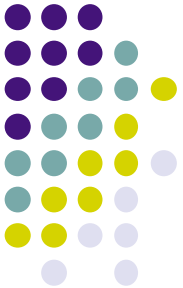


- Four we compare the expected and observed numbers and we compute for each cell (case) $(O_i-E_i)^2/E_i$

	Observed (O_i)		Expected (E_i)		
POS	**McCain'08**	**Obama'08**	**McCain'08**	**Obama'08**	**Percentage**
NN	33876	58550	30616	61734	41.6%
JJ	10677	18517	9715	19589	13.2%
VB	21927	54268	25244	50901	34.3%
RB	7117	17064	8022	16175	10.9%
Total	73,597	148,399	73,597	148,399	100%

For Obama and nouns, we have $((58{,}550 - 61{,}734)^2 / 61{,}734) = 164.22$. If H_0 is (really) true, such differences must be small.

Test Chi-Square

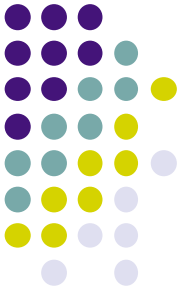


- For each cell (case), we compute the square of the difference divided by the expected number. We sum all these values.

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

	McCain'08	Obama'08
NN	347.05	164.22
JJ	95.30	58.63
VB	435.79	222.74
RB	102.11	48.81
Total	980.25	494.39
	χ^2 =	**1474.64**

Test Chi-Square



- Fifth, the decision

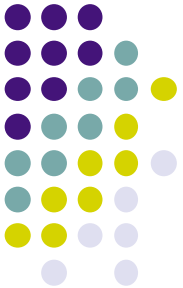
  The values for our χ^2 value is 1474.64

  Is this value large? Maybe too large if we admit that H_0 is true. How can we "objectively" say "it is too large"?

  Compare this (computed) value with the maximum value we may expected if H_0 is true…

- In fact we must admit an error in our test. Because even rare event has a (very) small probability (that is not null). Thus we must define the value (limit) for which 95% of the observations have a lower value…

Test Chi-Square



- We usually prefer specifying that the error α = 5% (significant level 1-α = 95%).
- Second point: The χ^2 is a family of distribution (we have more than one such distribution) and to specify which member of this family we need, we specify the number of *degree of freedom* (*dof*) which is $(r\text{-}1)\cdot(c\text{-}1)$

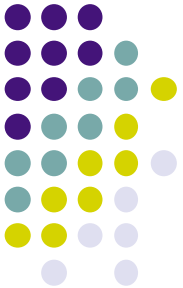
  This corresponds to the number of rows (r) and the number of columns (c) of our data (ignoring the total and percentage column or row)

Test Chi-Square

- Limits of the χ^2 distribution
- In our example, we obtain an observed value of 1474.64.
- The number of *dof* is $(4-1)\cdot(2-1) = 3$
- If H_0 is true, we may expect having value as large as 7.81 (α = 5%) or 11.3 (α = 1%)

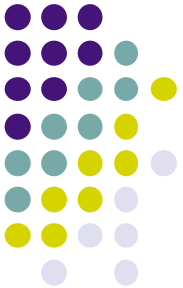
χ^2 dof	95%	99%
1	3.84	6.63
2	5.99	9.21
3	7.81	11.3
4	9.49	13.3
5	11.1	15.1
6	12.6	16.8
7	14.1	18.5
8	15.5	20.1
9	16.9	21.7
10	18.3	23.2

Test Chi-Square



- If H_0 is true, we may expect having value as large as 7.81 (with α = 5%) or 11.3 (with α = 1%)
- The observed value (1474.64) is larger than this limit (one-tail test) because we consider (to reject H_0) only one tail of the underlying distribution.
- Reject H_0 (no difference between the two distributions) and we accept H_1 (there is a significant difference)
- Where?

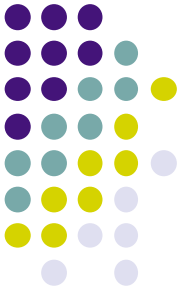
Test Chi-Square



- The main differences

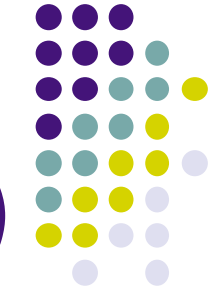
	Observed (O_i)		Expected (E_i)	
POS	**McCain'08**	**Obama'08**	**McCain'08**	**Obama'08**
NN	*33876*	58550	30616	61734
JJ	*10677*	18517	9715	19589
VB	21927	*54268*	25244	50901
RB	7117	*17064*	8022	16175

Test Chi-Square



- We must reject H_0 and thus accept H_1
  (there is a significant difference)
- Where?
  Obama uses more VB & RB, McCain more NN & JJ
- Why?
  Discourse analysis & political consideration …
  Buzzwords of the campaign
    "Country first: Reform, prosperity, peace"
    "Yes we can" or "change we believe in"
- Caution: the POS tagger is not perfect!

Test Chi-Square (2nd application)



- And for the distribution of the word type “Bush” in McCain’s speeches in 2008?

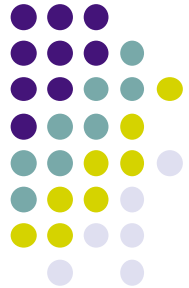
	Observed		Expected	
	S	**C-**	**S**	**C-**
“Bush”	26	398	104	320
not “Bush”	154,339	474,331	154261	474409

- Computing the difference between the observed and expected values according to the formula

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

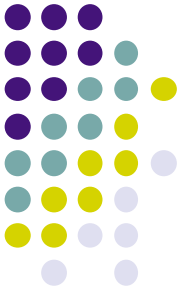
and we obtain χ^2 =78.13

Test Chi-Square (2nd application)



- Is this difference (χ^2 =78.13) large? Too large?
- Compared with the values in the table of the χ^2 under *dof* = (*r*-1)·(*c*-1) = 1·1 = 1
- If H_0 is true, we may expect having value as large as 3.84 (with α = 5%) or 6.63 (with α = 1%)
- The computed value χ^2 is large than the limit.
  The word type “Bush” in McCain’s speeches in 2008 does not follow the distribution of the US electoral speeches.
  McCain uses less often this name than Obama.

Our First Dice

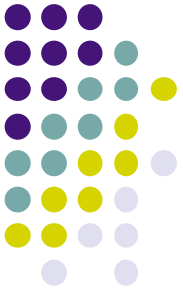


- With our dice we have observed the following values

	Observed	Expected	$(O_i - E_i)$	$(O_i-E_i)^2/E_i$
1	0	20/6	-3.33	3.33
2	5	20/6	1.67	0.83
3	2	20/6	-1.33	0.53
4	4	20/6	0.67	0.13
5	1	20/6	-2.33	1.63
6	8	20/6	4.67	6.53
			sum	13

- The computed χ^2 = 13.0 (with 5 *dof*).
  In the table with α=1% we have 15.1 (or 11.1 with α=5%)

Our Second Dice

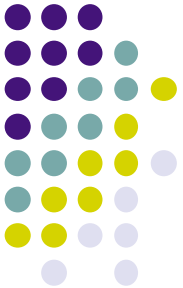


- With our dice we have observed the following values

	Observed	Expected	$(O_i - E_i)$	$(O_i-E_i)^2/E_i$
1	3	20/6	-0.33	0.33
2	5	20/6	1.67	0.83
3	3	20/6	-0.33	0.03
4	5	20/6	1.67	0.83
5	2	20/6	-1.33	0.53
6	2	20/6	1.33	0.53
			sum	2.8

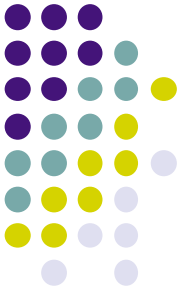
- The computed $\chi^2 = 2.8$ (with 5 *dof*).
  In the table with α=1% we have 15.1 (or 11.1 with α=5%)

Limit of the Chi-Square Test



- For each cell, the expected count must be 5 or greater. To avoid multiple cells with low count and thus we can increase (artificially) the χ^2 values.
- In studying word frequency, this constraint limits the application of this test to word occurring 5 times or more.
- For a lexical analysis, many word types will not be considered (Zipf's law)

Word Types Distribution

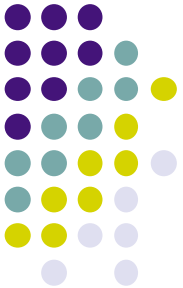


- Distribution of word types in the low frequencies classes
- Number of word types: 7663 (Obama'08), 7792 (McCain'08)

Frequency	Obama'08		McCain'08	
1	2573	33.6%	2958	38.0%
2	1042	13.6%	1112	14.3%
3	556	7.3%	641	8.2%
4	446	5.8%	435	5.6%
5	308	4.0%	313	4.0%

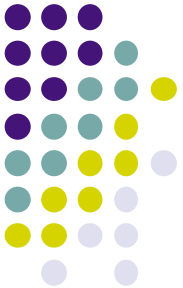
For the US corpus, this reduction is from 7,663 to 3,046 (or to 39.8% of the word types) for Obama 2008 and from 7,792 to 2,646 (7792-5146) (or 34%) for McCain 2008.

View/Verify the Context



- Finding pertinent (significant) features is the first step
- Explaining such phenomena is the second step
- Usually it is important to see the context
  and again the computer science may help
- How?
  KWIC
  + Perl script to specify multiple constraints in selecting words / contexts / sentences

KWIC Keyword In Context

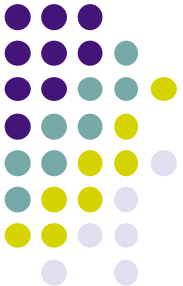


- Besides counting linguistic phenomena, computer science may provide other useful tools
- *KWIC* is such an example
- Provide the left and right context (number of words, number of characters) of a given word (exact spelling)
- Can be used to see the context around a term
- Example:
  Translation of “fort” (JJ) into the English language
  by “strong” or “powerful”
  “un fort orage”, “un café fort”, “un médicament fort”

Context around “Strong”

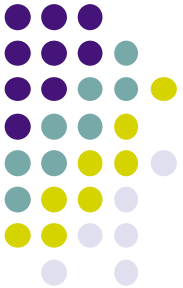
s pointed toward the December report as	strong	evidence of the long-awaited reversal in the nation's
5.8 billion Canadian dollars largely on	strong	foreign sales of forest products. *E* *S* However,
, and basically a black school that was	strong	in academics, "Dade said. *E* *S* "Before, we
finishing third in Iowa, maintened a	strong	lead in New Hampshire - but he no longer had the huge
etts Gov. Michael Dukakis maintened a	strong	lead in the Democratic race. *E* *S* ABC reported he
S* In both polls, Dukakis maintened a	strong	lead in the Democratic race. .End of Discourse *E* *
Er whose poll you're looking at - and a	strong	one, too, "said Jeff Alderman, chief of polling
Port on the seacost. *E* *S* Kemp, a	strong	proponent of states rights, has asked federal regu
rsuit of peace, NATO must soon offer a	strong	proposal on conventional and chemical weapons control
rsuit of peace, NATO must soon offer a	strong	proposal on conventional and chemical weapons control
ri Dubini Friday morning to "lodge a	strong	protest. *E* *S* "Defense Secretary Franl C. Carl
er Alexander Bessmertnykh read him a "	strong	protest. *E* *S* "The Soviet side cannot but view
the administration immediately lodged a	strong	protest with the Soviet ambassador here, saying the

Context around “Powerful”



ted. *E* *S* It also said two other "	powerful	bombs" were defused "in the last several days"
ederation of Economic Organizations, a	powerful	business alliance, is planning a leap into the 21s
itian army Col. Jean-Claude Paul, the	powerful	commander of the key batallion in Port-au-Prince,
. *E* *S* Despite the existence of two	powerful	drugs to treat the rare form of pneumonia, scienti
and simulated windsurfing in front of a	powerful	fan. *E* *S* Among the poeple wearing shorts were
nd West Germany, both with politically	powerful	farming lobbies, have sought an increase of $3.1 b
till was a land of barbarian tribes and	powerful	feudal warriors - one of Japan's last frontiers. *
out. *E* *S* "It's a vera silent but	powerful	force in Southern politics, "Rose said. *E* *S*
en. *E* *S* The reflex is particulary	powerful	in children, doctors say. *E* *S* Kendall was in
en. *E* *S* The reflex is particulary	powerful	in children, doctors say. *E* *S* Tecklenburg sai
ficient in the short-term, it provides	powerful	incentive for workers to sabotage innovative techno
eight straight term. *E* *S* With the	powerful	infrastructure of the governing Colorado Party at h
k was retained as head of South Korea's	powerful	intelligence agency, the Agency for National Secur
hn Moo-hyuk was retained as head of the	powerful	intelligence organization, the Agency for National

Strong vs. Powerful



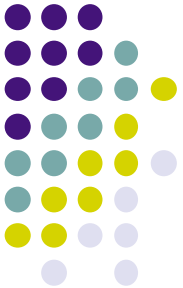
- Are you drinking a “strong coffee” or a “powerful coffee”?
- Are you working with a “strong PC” or a “powerful PC”?
- Given the context, the translation could be “strong” or “powerful” (but the distinction is not always (for a computer at least) very clear, e.g., “strong/powerful drug”)

- Based on newspaper articles, we can find

Strong vs. Powerful

C(w)	C(strong w)	C(powerful w)	w
3418	4	13	force
933	0	10	computers
2337	0	8	computer
588	0	6	machines
2266	0	5	Germany
3745	0	5	nation
3685	50	0	support
3616	58	7	enough
3741	21	0	sales
1093	19	1	opposition
802	18	1	showing
2501	14	0	defense

Conclusion



- Statistical tests could be useful to verify a theory
- The interpretation and explanation of the underlying phenomenon are not included in the test!
- The Chi-square test could be used in various contexts
- But
  - random sampling
  - it needs at least 5 (expected) observations in each cell.